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TECHNICAL REPORT No.

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V-DINUCLEONS

by

H. PRIMAKOFF

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"V Dinucleons"*

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(Received December 21, 1953)

In a previous paper[1] (hereafter referred to as A) we have suggested the possibility of the existence of "V deuterons": particles involving a nuclearly stable structure of a proton bound to a neutral V.[2] These V deuterons are subject to nonmesonic and mesonic decay according to the schemes:

$$V\text{ deuteron}\equiv[V^0+p]\rightarrow\begin{cases} p+n+\approx 175\text{-Mev K.E.} & (1)\\ p+p+\pi^-+\approx 35\text{-Mev K.E.} & (2)\\ p+n+\pi^0+\approx 35\text{-Mev K.E.} & (3)\\ d+\pi^0+\approx 35\text{-Mev K.E.} & (3')\\ n+n+\pi^++\approx 35\text{-Mev K.E.} & (4)\end{cases}$$

Schemes (3′) and (4) were not mentioned in A; on the basis of charge symmetry, scheme (4) should obviously occur if scheme (2) occurs, while scheme (3′) is an alternative to (3) since the n and p may on occasion be produced in a bound (deuteron) state. The relative probabilities of the nonmesonic decay scheme (1) and of the mesonic decay schemes (2), (3), (3′), (4) have been shown in A to be of the same order of magnitude; the mean life of the V deuteron has been shown in A to be of the same order as the mean life (for the mesonic decay) of a free V^0 ($\approx 3\times 10^{-10}$ sec).

We now wish to point out that an alternative structure of the V deuteron is also possible, namely $[V^++n]$, i.e., a nuclearly stable structure of a neutron bound to positively charged V; this alternative V deuteron will also be capable of decay according to schemes (1) through (4) but with a considerably larger kinetic energy release, since the rest energy of the $V^\pm$ is some 85 ± 40 Mev larger than the rest energy of the V^0[$M(V^0)\approx(2185\pm 10)m_e$; $M(V^\pm)\approx(2350\pm 80)m_e$].[3]

In addition, particles which might be called "negative V deuterons," "V diprotons," and "V dineutrons" can be nuclearly stable even if the "specifically nuclear" attractive forces between nucleons and V's in various angular momentum states are no stronger than the corresponding forces between nucleons and other nucleons. For unlike ordinary dinucleons, all of these "V dinucleons" can exist in $^3S_1$ states (Pauli exclusion inoperative between a V and a nucleon); moreover the relatively greater mass of the V will tend to increase the binding energy of a V dinucleon relative to that of a deuteron. Having postulated the existence of these particles, one must envisage the following possible mesonic and nonmesonic decay schemes:

$$\text{negative } V\text{ deuteron}\equiv[V^-+n]\rightarrow n+n+\pi^- \qquad (5)$$

$$V\text{ diproton}\equiv[V^++p]\rightarrow\begin{cases} p+p & (6)\\ p+n+\pi^+ & (7)\\ d+\pi^+ & (7')\\ p+p+\pi^0 & (8)\end{cases}$$

$$V\text{ dineutron}\rightarrow\begin{cases}\equiv[V^0+n]\rightarrow\begin{cases} n+n & (9)\\ n+p+\pi^- & (10)\\ d+\pi^- & (10')\\ n+n+\pi^0 & (11)\end{cases}\\ \equiv[V^-+p]\rightarrow\begin{cases} n+n & (12)\\ n+p+\pi^- & (13)\\ d+\pi^- & (13')\\ n+n+\pi^0 & (14)\end{cases}\end{cases}$$

The mean lives of these particles will again be $\approx 10^{-10}$ sec [assuming comparable mean lives (for the mesonic decay) of the free V^0, the free V^+ and the free V^-] while the branching ratios for the mesonic and nonmesonic decays will, as before, be of the order of unity.[1]

It is very interesting to note that the mesonic decay of a V dineutron according to scheme (13′) will appear in an emulsion or a cloud chamber as the two-body decay into a deuteron and a π^- meson of a neutral particle with mass. $[\{2350\pm 80\}+\{1836\}-[V^-+p]\text{ binding energy}]m_e\approx\{4180\pm 80\}m_e$.[4] A decay of just this type has recently been reported by Lal, Pal, and Peters[5] who estimate the mass of the neutral particle as $\{4120\pm 20\}m_e$.

It should also be mentioned, as pointed out to us by Bolsterli, that neutral particle→visible $\{p+\pi^-\}$ decays, with a Q (calculated on a two-body basis) considerably different from (e.g., very much larger than), 35–40 Mev,[6] might actually be V-dineutron decays according to schemes (13) or (10). On such an assumption, these anomalous Q values would form a distribution with an upper limit given by the Q of the V^- decay (≈ 125 Mev).

We wish to thank Professor R. D. Sard for a helpful discussion.

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] W. Cheston and H. Primakoff, Phys. Rev. 92, 1537 (1953).

[2] According to a notation now coming into use: V^0, V^+, $V^-\equiv\Lambda^0$, Λ^+, $\Lambda^-\equiv$ neutral, positive, negative hyperon.

[3] We take 38 Mev for the Q value of $V^0\rightarrow p+\pi^-$, and suppose that $M(V^+)\cong M(V^-)$. Q values for the decay: $V^\pm\rightarrow$ neutron $+\pi^\pm$ may be obtained from Lal, Pal, and Peters, Phys. Rev. 92, 438 (1953). ($Q=135\pm 35$ Mev); King, Seeman, and Shapiro, Phys. Rev. 92, 838 (1953). ($Q=103\pm 20$ Mev); M. Ceccarelli and M. Merlin, Nuovo cimento 10, 1207 (1953) ($Q=131\pm 24$ Mev). We also suppose that the V^+ possesses the alternative decay scheme: $V^+\rightarrow$ proton $+\pi^0$ [with a Q, according to Bonetti, Settl, Panetti, and Tomasini, Nuovo cimento 10, 1736 (1953), of 115.3 Mev].

[4] We take the $[V^-+p]$ binding energy as ≈ 3 Mev; assumption of the scheme (10′) would predict a mass of $\{4015\pm 10\}m_e$ for the neutral particle, provided that the $[V^0+n]$ binding energy is also taken as ≈ 3 Mev. We also suppose that the rates of the energetically possible processes: $[V^-+p]\rightarrow V^0+n$ and $[V^++n]\rightarrow V^0+p$ are not appreciably greater than $\approx 10^{10}\text{ sec}^{-1}$.

[5] Lal, Pal, and Peters (reference 3).

[6] Leighton, Wanlass, and Anderson [Phys. Rev. 89, 148 (1953)] find, in decays of neutral particles into $p+\pi^-$, Q values ranging from 10 ± 3 Mev to 87 ± 15 Mev; it is likely that at least several of these anomalous Q's are not consequences of experimental error.

CONCENTRATION OF A CYCLOTRON BEAM BY STRONG FOCUSING LENSES

by

F. B. SHULL, C. E. McFARLAND

and M. M. BRETSCHER

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Concentration of a Cyclotron Beam by Strong Focusing Lenses*

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(Received July 3, 1953)

A pair of strong focusing magnetic lenses has been used to concentrate a part of the external beam from the Washington University cyclotron. The lenses focus the beam at distances up to more than four meters beyond the end of the cyclotron. This produces approximately an eightfold improvement in beam intensity.

INTRODUCTION

THE Department of Physics at Washington University operates a 45-in. fixed frequency cyclotron which produces 10 Mev deuterons, 20 Mev alpha particles, or 5 Mev protons (by accelerating molecular hydrogen ions). The deuteron beam is used most commonly, and external currents of 200 microamperes and higher are easily maintained. The cyclotron has been used extensively to produce radioactive isotopes; in addition, a number of attempts have been made to use its external beam to investigate nuclear reactions.

The cyclotron is inherently less well suited for studying nuclear reactions than the Van de Graaf generator. For maximum utility in such studies, a stream of projectile particles is desired which is sufficiently intense, well collimated, highly monoenergetic, and preferably variable in energy in an easily controlled way. Moreover, the experimental area should be as free as possible from undesirable background radiations. The Van de Graaff generator fulfills all of these requirements; its only drawback is that the maximum available projectile energy is rather low, up to 4 or 5 Mev, and can be increased only by the expenditure of considerable effort.

By contrast, the cyclotron produces a generous external beam of fairly high energy, which spreads out in a wide diverging fan as it leaves the machine. Radiation background is high and extensive shielding is mandatory. Because the beam fans out so much, the beam intensity is actually very low at distances far enough away that the background is low. The particle energy is not very well defined (for 10 Mev deuterons a spread in energy of 40 to 60 kev is common), and it is not susceptible to variation in any simple, easy way.

A number of successful attempts have been made to overcome these shortcomings of the cyclotron as a tool in nuclear reaction studies. At the University of Rochester[1] a single electromagnet is used to bend and focus the external beam, following the method proposed by Cross.[2] At the University of Pittsburgh,[3] a very elaborate system of successive electromagnets concentrates and analyzes the beam, and it is possible to achieve an appreciable narrowing of the energy spread to as little as 10 kev for 8 Mev protons. Such arrangements as these are, unfortunately, not particularly feasible for use with the Washington University cyclotron. The chief reason is that the machine is housed in a regrettably small underground room, with no space available for expansion. It would be difficult to squeeze in large heavy electromagnets plus adequate shielding and still have space for research apparatus.

The recent proposals by Courant, Livingston, and Snyder[4] to focus charged-particle beams with hyperbolic magnetic fields have opened up an attractive new avenue of approach to the problem. They pointed out that a sequence of pairs of quadrupole magnets, with hyperbolic pole faces, act as lenses which can be arranged so as to concentrate a beam of charged particles without changing the general direction of the beam. This notion is particularly appropriate to the problems of the Washington University cyclotron, since, by sheer good fortune, the maximum available space lies in the direction in which the beam is already heading. Such magnetic lenses offer the attractive possibility of increased intensity, brought about by compact and inexpensive equipment. To be sure, no improvement of energy spread will result, nor will the beam energy be variable. Still, the prospect of an adequately intense external beam, brought to focus at a point far away from the cyclotron and from the worst of the background radiation, represents a worthwhile and attainable goal. Accordingly, we have designed, built, and tested a hyperbolic lens pair for this purpose, with successful and satisfying results. Recently Cork and Zajec[5] have reported on a similar project, using two lens pairs, for the 60-in. cyclotron at Berkeley.

EQUIPMENT

A photograph of one of the magnet cores is shown in Fig. 1. It is fashioned of mild steel, has an external diameter of 25 cm and length of 10 cm, and the separation between opposite pole tips is 4.0 cm. Each core is made in quadrants which are strapped together by a

* Assisted by the joint program of the U. S. Office of Naval Research, and the U. S. Atomic Energy Commission.
[1] D. A. Bromley, private conversation.
[2] W. G. Cross, Rev. Sci. Instr. 22, 717 (1951).
[3] Precision Scattering Project, Report No. 2, (May, 1952) Scaife Radiation Laboratory, University of Pittsburgh.
[4] Courant, Livingston, and Snyder, Phys. Rev. 88, 1190 (1952).
[5] B. Cork and E. Zajec, Phys. Rev. 92, 853(A) (1953).

surrounding iron band. Steel pins are used to insure proper relative positioning of the quadrants.

A completely assembled magnet is shown in Fig. 2. Each of the four exciting coils has 600 turns of No. 14 wire with Formvar insulation. Copper tubing for water-cooling was buried within each coil as it was wound. Power is supplied by a dc generator to the four coils in series, and currents in the range from two to six amperes are commonly required. The power requirement for one magnet is about 300 watts at six amperes when the coils are warm. The two magnets are supplied in parallel from a single generator, with separate control resistors and ammeters.

The magnetic field in the pole gap was measured with a small flip-coil and ballistic galvanometer over a wide range of excitation currents. The hyperbolic pattern of the lines of force is quite rigorously maintained even with currents as high as 10 amperes, and saturation of the core is barely noticeable at nine amperes.

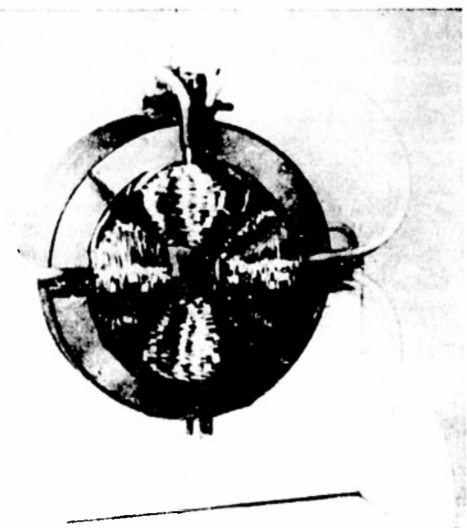

FIG. 2. Completely assembled magnet.

TEST ARRANGEMENT

The cyclotron beam, after it passes the exit strip (septum) on the dee and is peeled away by the deflector and by the decreasing magnetic field, executes a kind of graceful unwinding spiral path, while at the same time fanning out rather widely in the horizontal plane. Some of the beam, if allowed to run free, would encounter the cyclotron magnet and be stopped, and is therefore not useful. The remainder would skirt past the corner of the cyclotron magnet into a region which is essentially field-free. Here the unwinding spiral orbits degenerate into a family of diverging straight lines which, if extrapolated backward, would appear to intersect approximately at a point. The magnetic lenses were placed in this field-free region, as close as possible to the outside corner of the cyclotron magnet. The apparent point of intersection of the diverging straight-line orbits is thus the effective "object" for the lens system. A small portion of the diverging beam is conducted through evacuated tubing to the lens pair, where a still smaller portion (2.5 cm wide and 1.25 cm high) is permitted to enter and pass through the lenses. The evacuated tubing continues through and beyond the lenses; it was terminated alternatively with either (1) a quartz plate, to observe the beam pattern by fluorescence, or (2) an insulated Faraday cup, connected to ground through a sensitive galvanometer, to measure beam currents. The Faraday cup was immediately preceded by an aperture of variable size so that current readings could be interpreted in terms of beam intensity.

FIG. 1. Quadrupole magnet core, with hyperbolic pole faces.

TESTS WITH A SINGLE LENS

The first tests were preliminary in nature, to study the focusing properties of a single quadrupole lens. A single lens will converge the beam in one dimension, while diverging it in the other. The lens current was adjusted to produce a sharp line focus on the quartz plate at each of several different distances Q beyond the lens. When $1/Q$ is plotted as a function of the current which produces the line focus, a straight line with positive slope results (Fig. 3). When the straight line is extrapolated backward to its intersection with the $1/Q$ axis, the intercept gives the negative inverse of the "object" distance. In this way, the "object" was found to be approximately 235 cm in front of the single lens. In later tests using two lenses, the first lens of the pair always occupied this same position relative to the "object."

Figure 3 may also be used to determine the focal length of a single lens as a function of current. Ordinates measured from the above-mentioned intercept with the $1/Q$ axis yield the inverse of the focal length.

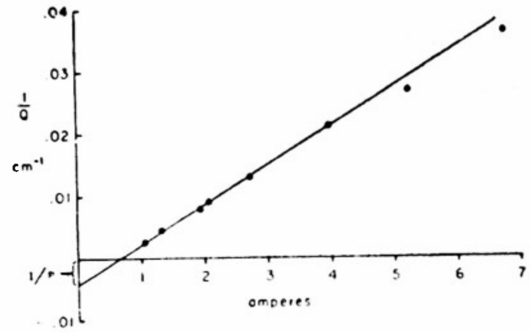

FIG. 3. Single-lens test. P is the "object" distance. See text for details.

Single-lens tests were performed with each lens separately in the same position, with perfectly consistent results.

TESTS WITH TWO LENSES

The behavior of a quadrupole lens pair is a function of five parameters. These are (1) the "object" distance, (2) the separation distance between lenses, (3) the image distance, (4) the current in the first lens, and (5) the current in the second lens. Simultaneous focusing in two dimensions (double focusing) is possible for an infinite number of combinations of values for these five parameters; but, if any three are fixed, the other two are determined. In our tests, it was convenient to fix the first three parameters in various suitable combinations, and, for each such combination, to experiment with the two currents to find their best values for double focusing. In practice, the "object" distance was always 235 cm, the separation distance was either 30 or 120 cm, and the image distance (measured from the second lens) was varied over a range from 77 to 411 cm.

In general, the lens pair acts like an astigmatic optical lens; it produces simultaneously both a vertical and horizontal line image at different distances. This behavior affords a convenient but tedious method for adjusting the two currents to obtain double focusing. While one current is held constant, the other is varied, and the beam pattern is observed on the quartz plate. As the second current is varied, the beam pattern passes successively through a vertical and horizontal line focus. This procedure may be repeated with successively higher values of the first current; the two line foci move closer together, eventually merge, and finally move apart but in reversed sequence. The results are illustrated in Fig. 4; the abscissa i_1 is the first current, held constant during each step of the procedure; the ordinate is the variable current i_2. Curve H gives values of i_2 which produce horizontal line images, and curve V gives vertical line images. Their intersection gives the double focusing currents I_1 and I_2.

This routine was followed for each of a number of different image distances Q, and double focusing currents were determined for each. Figure 5 shows how Q is related to the double focusing currents I_1 (for the first lens), and I_2 (for the second lens).

CHARACTERISTICS OF THE DOUBLE FOCUSED BEAM

In our tests, a rectangular portion of the cyclotron beam, 2.5 cm wide by 1.25 cm high, was allowed to enter the first lens. The first lens acts to decrease the width and increase the height; the second lens does the reverse. The double focused image is approximately round, but can be made oblong by minor manipulation of the currents. Various characteristics and peculiarities of the double focused beam are summarized as follows:

(1) When the positions of the first lens and of the quartz plate are held constant and the separation distance between lenses is increased from 30 cm to 120 cm, the area of the double focused image decreases by as much as a factor of two. This did not, however, appear to lead to increased intensity at the focus point, which suggests that some of the beam was intercepted between the magnets. Since compactness seemed desirable, all of the results discussed in the following paragraphs apply to a 30-cm separation distance.

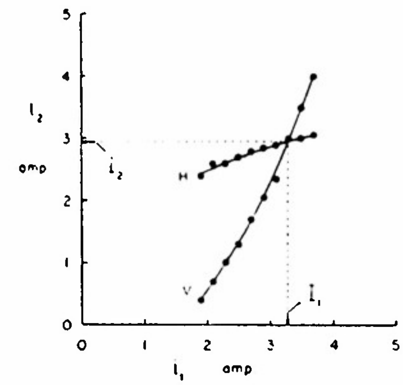

FIG. 4. Procedure for finding magnet currents for double focusing at $Q=176$ cm. Magnets 30 cm apart. See text for explanation.

(2) The size of the image increases as the image distance increases. In this respect, the magnetic lenses are similar to optical lenses. At $Q=411$ cm the image diameter is slightly greater than 13 mm. At $Q=176$ cm the diameter is approximately 6 mm, and this is about the smallest image size we were able to obtain.

(3) The quadrupole lens pair suffers from second-order defocusing effects which are analogous to spherical abberration in optical lenses. At $Q=411$ cm, the image diameter drops from 13 mm to approximately 9 mm if the initial 2.5 cm by 1.25 cm aperture is replaced by a 9.5-mm hole. However, the image intensity falls off badly when this is done.

(4) The beam intensity is not constant over the whole area of the focused image. We tested this at

$Q=411$ cm by placi[ng] apertures of different diameters at the focus point, just ahead of the Faraday cup. The resulting variations of the current to the cup suggest that the intensity might vary as a function of radius (measured from the center of the circular image) approximately as shown in Fig. 6. We are by no means certain, however, that the intensity is a function of radius only.

(5) The use of the lenses brings about an average increase of beam intensity by a factor of about eight; this figure was arrived at by measuring the Faraday cup current (through various limiting apertures), both with and without current in the lens pair. This should be compared with the factor of 30 reported by Cork and Zajec.[5] The difference can probably be explained by these facts: (a) Cork and Zajec used two lens pairs instead of one, and (b) their lens aperture was 5 cm compared with our 4 cm. Thus, for example, the first pair might be used to render the beam parallel; the object distance for the second pair is then infinite,

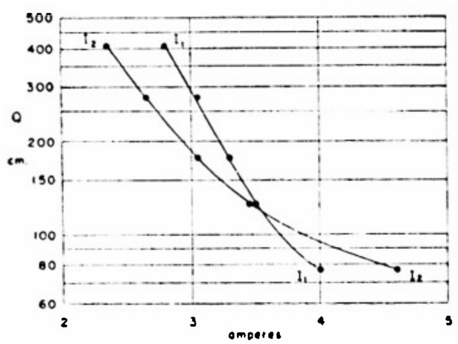

FIG. 5. Magnet currents which produce doubly focused image at various distances Q beyond second magnet. Magnets 30 cm apart.

which leads to small magnification and a concentrated image.

(6) It is our experience so far that the focused beam intensity and current are critically sensitive to rather minor adjustments in the tuning of the cyclotron. Small changes of the deflector voltage or, to a lesser extent, of the cyclotron field bring forth drastic and erratic changes in the Faraday cup current, even

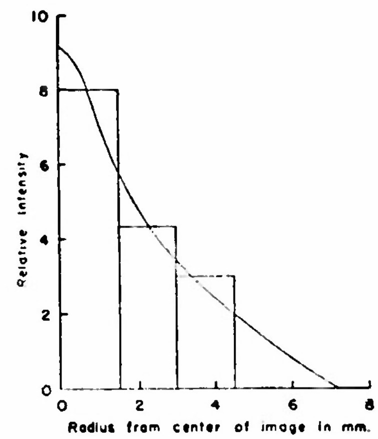

FIG. 6. Approximate variation of intensity in doubly focused image at $Q=411$ cm, assuming intensity depends on radius only. Magnets 30 cm apart.

though the total external cyclotron beam is very little changed thereby. It is essential that the cyclotron operator should follow the fluctuations of the Faraday cup current and continually retune the machine for maximum current.

Moreover, the Faraday cup current responds in a peculiar and somewhat disheartening way when we increase the total external cyclotron beam by boosting the power input to the machine. For example, with the cyclotron giving out a 60 microampere total beam, we observe a Faraday cup current of 0.65 microampere through a 3 mm aperture at the focus point ($Q=176$ cm). If we then double or triple the total beam, the Faraday cup current may respond sluggishly with an increase by a factor of no better than perhaps 1.2 to 1.5.

Such flighty behavior is probably connected with changes in the position and configuration of the "object" of the lens pair. It is to be hoped that our understanding and control of these phenomena will improve as our experience grows.

We are indebted to Mr. Otto Retzloff for his skillful and patient fabrication of the magnetic lenses and associated equipment, and to Mr. A. A. Schulke, Mr. A. B. Phillips, and the rest of the cyclotron crew for the valuable assistance which they so generously gave.

PHOTON SPLITTING

IN A

NUCLEAR ELECTROSTATIC FIELD

by

M. BOLSTERLI

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Photon Splitting in a Nuclear Electrostatic Field*

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(Received December 18, 1953)

The cross section for the splitting of a photon into two photons in a nuclear electrostatic field has been calculated from the vacuum polarization Hamiltonian of Euler and others to first order in $e^2/\hbar c$ for low-energy incident photons ($p < mc$). For a favorable experimental case, photons of energy 840 kev incident on lead with antiparallel product photons each emitted at 90° relative to the incident photon, the cross section is 2.3×10^{-33} cm²/sterad².

THE nonlinear terms in the Maxwell equations arising from the polarization of the vacuum[1] result in several interesting effects: coherent photon scattering by a nuclear electrostatic field,[2] scattering of photons by photons,[3] and photon splitting into two product photons in a nuclear electrostatic field, the last briefly discussed by Williams.[4] The cross sections for the first two processes are quite small and hard to verify experimentally, the first because of the difficulty in separating out the coherent scattering by the nuclear electrostatic field from other coherent nuclear and atomic-electron photon scatterings (however, see Wilson[5]), the second because of the lack of gamma-ray sources capable of furnishing enough photons. The cross section for the third process is no less small, but strong gamma-ray sources are available, high-Z nuclei furnish relatively large electrostatic fields, and energy discrimination can be made to eliminate unwanted inelastic scatterings which act as a background for the splitting process.

As calculated in the following, the cross section for the production of two photons oppositely directed and perpendicular to an original photon of energy 1.65 (in units of mc^2—0.84-Mev γ of Mn[54]) incident on a nucleus of charge $Z=82$ is 2.3×10^{-33} cm²/sterad². (We use energy units of mc^2 throughout.) Of the product photons, nearly all have energy between 0.4 and 1.3

* Assisted by the joint program of the U. S. Office of Naval Research and U. S. Atomic Energy Commission.

† Holder of Shell Fellowship 1952-1954.

[1] H. Euler and B. Kockel, Naturwiss. 23, 346 (1935); W. Heisenberg and H. Euler, Z. Physik 98, 714 (1936); H. Euler, Ann. Physik. 26, 398 (1936); V. Weisskopf, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 14, No. 6 (1936).

[2] M. Delbrück, Z. Physik 84, 144 (1933); A. Achieser and I. Pomerantschuk, Physik. Z. Sowjetunion 11, 478 (1937); N. Kemmer, Helv. Phys. Acta 10, 112 (1937); N. Kemmer and G. Ludwig, Helv. Phys. Acta 10, 182 (1937); F. Rohrlich and R. Gluckstern, Phys. Rev. 86, 1 (1952); H. A. Bethe and F. Rohrlich, Phys. Rev. 86, 10 (1952).

[3] O. Halpern, Phys. Rev. 44, 855 (1933); H. Euler and B. Kockel, reference 1; H. Euler, reference 1; R. Karplus and M. Neuman, Phys. Rev. 80, 380 (1950) and 83, 776 (1951).

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[5] R. R. Wilson, Phys. Rev. 90, 720 (1953).

due to a factor $k_1^3k_2^3$ (a continuous range of energies for each of the two scattered photons is allowed by the conservation laws: $\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2+\mathbf{K}$ and $k=k_1+k_2+K^2/2M \approx k_1+k_2$). In the case of double Compton scattering from electrons[6] (a competing process with a cross section per atom very roughly $(2\pi^2)^{-1}\alpha(e^2/mc^2)^2Z=2.4\times10^{-27}$ cm^2/sterad^2), for the same configuration, the conservation equations for the energies of the two product photons have the form $\mathbf{k}=\mathbf{K}$, $\mathbf{k}_1=\mathbf{k}_2$, and $k+1=k_1+k_2+(K^2+1)^{\frac{1}{2}}$. For $k=1.65$, these give $k_1+k_2=0.72$. The energy of a single-Compton scattered photon under these circumstances is 0.62 (random coincidences might compete here because of the relatively huge cross section). The cross section for double Compton scattering from nuclei is relatively negligible (approximately $(2\pi^2)^{-1}Z^2\alpha[(Ze)^2/AMc^2]^2[kmc^2/AMc^2]^2 \approx 10^{-37}$ cm^2/sterad^2). Thus, by biasing counters to accept only photons in coincidence with each photon energy lying between 0.7 and $1.65-0.7=0.95$ and requiring that the two energies add to 1.65, most of the background scattering should be eliminated; moreover, if photons are accepted only in this energy range, the cross section is still about 7×10^{-34} cm^2/sterad^2.

This cross section for the splitting of a photon into two photons has been calculated (in lowest order in α) for small incident momenta ($k<1$) from the equivalent vacuum polarization Hamiltonian of Euler and others:[7]

$$H_I=-(\alpha^2/360\pi^2)[(h/mc)^3/mc^2][(\mathbf{D}^2-\mathbf{B}^2)^2+7(\mathbf{D}\cdot\mathbf{B})^2]$$

of which the contributing terms are (nuclear electrostatic field $\mathbf{D}_n$, photon field $\mathbf{D}_p$, $\mathbf{B}_p$; $\mathbf{D}=\mathbf{D}_p+\mathbf{D}_n$, $\mathbf{B}=\mathbf{B}_p$):

$$-(\alpha^2/180\pi^2)[(h/mc)^3/mc^2] \times[2\mathbf{D}_p{}^2\mathbf{D}_p\cdot\mathbf{D}_n-2\mathbf{B}_p{}^2\mathbf{D}_p\cdot\mathbf{D}_n+7\mathbf{D}_p\cdot\mathbf{B}_p\mathbf{D}_n\cdot\mathbf{B}_p].$$

By using a shielded potential for the nucleus,

$$\mathbf{D}_n=-Ze\,\mathrm{grad}\{[\exp(-r/a)]/r\},$$

the differential cross section for splitting of the incident photon of momentum $\mathbf{k}$ into two photons of momenta $(\mathbf{k}_1,\mathbf{k}_1+d\mathbf{k}_1)$ in $d\Omega_1$, $\mathbf{k}_2$ in $d\Omega_2$, is obtained (wave-numbers in units of mc/h, a in units of h/mc):

$$\sigma dxd\Omega_1 d\Omega_2=\frac{\alpha^5(h/mc)^2Z^2k^6}{32\cdot81\cdot25\cdot\pi^4}\cdot\frac{x^3(1-x)^3(Ax^2+Bx+C)}{[\mathbf{K}^2/2k^2+1/2a^2k^2]^2}dxd\Omega_1d\Omega_2\text{ cm}^2,$$

$$\int_0^1\sigma dx=Qk^6Z^2\text{ cm}^2/\text{sterad}^2,$$

where $\mathbf{K}=\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2$, $k_2=k-k_1$, $x=k_1/k$, and A, B, C are functions of the angles involved, say, $\cos(\mathbf{k},\mathbf{k}_1)=\cos\theta_1=u$; $\cos(\mathbf{k},\mathbf{k}_2)=\cos\theta_2=v$; $\cos(\mathbf{k}_1,\mathbf{k}_2)=\cos\theta_{12}=w$. Then in terms of u, v, and $t=1-w$,

$$\mathbf{K}^2/2k^2=tx^2+x(v-u-t)+1-v,$$

and

$$\begin{aligned}A=\;&157t^3+t^2(-193+157v+157u\\&-139v^2-139u^2+157uv)+t(278-278v\\&-278u+157v^2+157u^2-36uv),\end{aligned}$$

$$\begin{aligned}B=\;&-157t^3+t^2(193-278v-36u+278v^2-157uv)\\&+t(-278+363v+193u-278v^2-36u^2\\&+36uv+157uv^2-157u^2v)+(v-u)(278\\&-278v-278u+157v^2+157u^2-36uv),\end{aligned}$$

$$\begin{aligned}C=\;&(1-v)\{t^2(18+139v)+t(85-121v-121u\\&+157uv)+139-139v+157v^2\\&+18u^2-314uv+139u^2v\}.\end{aligned}$$

For the particular case $\theta_1=\pi/2$, $\theta_2=\pi/2$, $\theta_{12}=\pi$, Q is found to be 1.68×10^{-38} cm^2/sterad^2. The variation of Qk^6Z^2 with, for example, deviations of θ_{12} from π for fixed $\theta_1=\pi/2$, $\theta_2=\pi/2$, is relatively slow.

If $k>10^{-2}$, A, B, and C are very small in the region where the term $1/a^2k^2$ becomes significant (i.e., as $\mathbf{K}^2/2k^2\to0$). Then a rough estimate of the total cross section $\sigma(k)$ is $(4\pi)^2$ times the differential cross section for the configuration above, and works out to be about $0.7\times10^{-2}\alpha^5(h/mc)^2Z^2k^6$ cm^2.

For high-energy incident photons ($k\gg1$), a simple calculation by the Weizsäcker-Williams method,[8] using the cross section for high-energy scattering of photons by photons derived by Achieser,[9] shows that

$$\sigma(k)=b\alpha^5(h/mc)^2Z^2\log(\eta k)\text{ cm}^2,$$

where η is of the order of 1; a similar result using a cruder estimate of the cross section for high-energy photon-photon scattering [$\sim\alpha^4(h/mc)^2$ instead of $\sim\alpha^4(h/mc)^2(1/k_1k_2)$] has been given by Williams.[4] If the expressions for small k and for large k are equated near $k=2$, then $b\approx1/2$.

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